Transmission Lines

(For those of a nervous disposition –
Not a lot of maths!)

Peter M0DCV

But, alas, still a little bit!

Topics

- History
- Lumped and Distributed Components
- Co-axial and Twin Feeders
- Electrical Model
- Characteristic Resistance
- Characteristic Impedance
- Waves Along a Line
- Wave Reflections and Standing Waves
- Line Impedance Variations
- Quarter Wave Transformer

- Demonstration

History

An unusual beginning in 1729 when Stephen Gray, a 63 year old pensioner in a charitable institution for elderly men, discovered that the electrostatic attraction of small bits of paper could occur at one end of a damp string several hundred feet long when an electro-statically charged body was touched to the end.

Sixty years earlier Otto Guericke von Magdeburg noted that short lengths of thread attached to a primitive electro-static machine became charged throughout their length. He concluded that “electric effluvia” were transmitted along the line.

History (Continued)

Gray established that electrical transmission occurred when the moist packthread was supported by dry, silk threads and not by fine, brass wire.

This distinction between conductors and insulators was developed in the succeeding five years by Charles DuFay. He and Benjamin Franklin reported the existence of positive and negative electricity.

History (Continued)

In 1753 the February 17th issue of the “Scots Magazine” had a proposed design for an electrical transmission system. It consisted of 26 parallel wires, one for each of the letters of the alphabet, supported at 60 foot intervals by glass insulators.

A sequence of letters could be sent by touching the end of a wire with a charged rod.

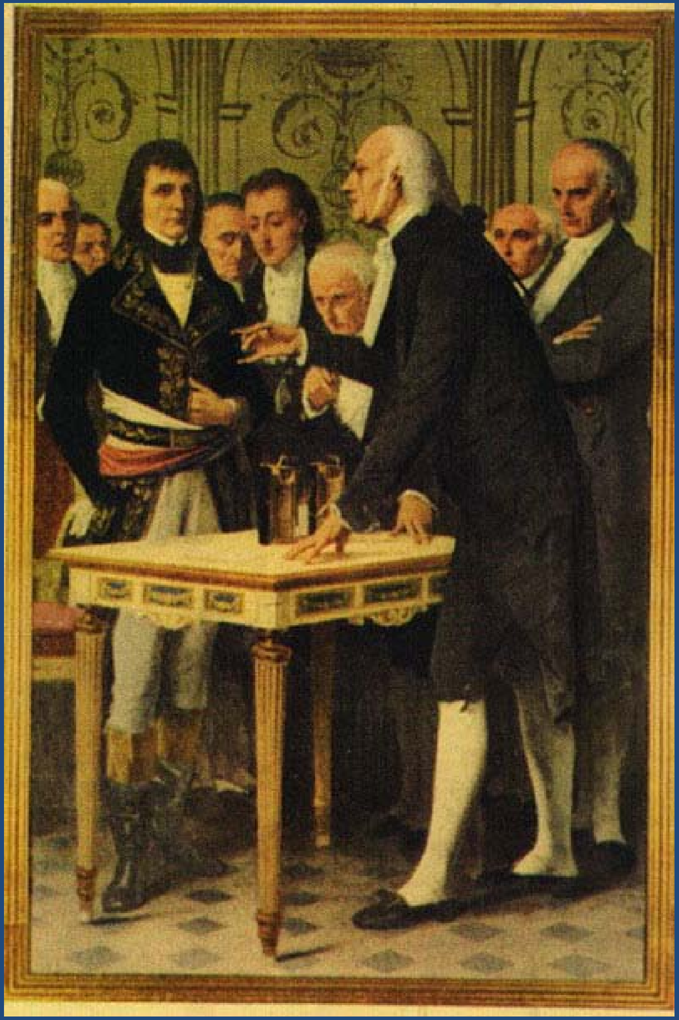
History (Continued)

Volta's discovery in 1800 of the chemical pile and Oersted's discovery of the magnetic effect of a current in 1820 resulted in the electric telegraphs of Gauss, Henry and others in the early 1830s.

These were followed by those of Wheatstone and Cook in 1839 and Morse in America in 1844.

In both cases buried transmission lines were used first but soon replaced by open wires on telegraph poles. By 1850 there were thousands of miles of telegraph transmission lines based on empirical design and construction.

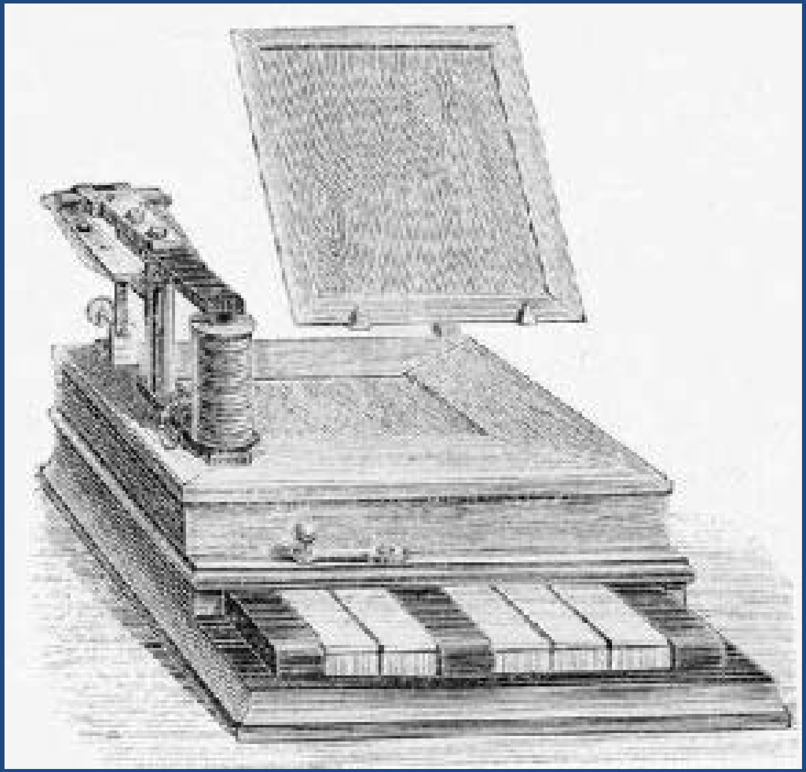
Volta



Volta's early piles.

Early Telegraphs

Morse – Using the Telegraph



Baudot's Telegraph

History (Continued)

The need for telegraphs to cross expanses of water led by the 1850s to lengths of over 300 miles of submarine cables.

The operation of these long underwater cables revealed a new phenomenom – that of signal distortion. The original “squareness” of the signal became blurred waverings on a jittery baseline.

History (Continued)

Sir William Thomson – later Lord Kelvin – undertook the first distributed circuit analysis of a uniform transmission line. He showed that transmission was a commercial possibility over transatlantic distances.

The world's first professional association for electrical engineers was founded as the "Society of Telegraph Engineers " in London in 1871 and became the IEE in 1889.

The invention of the telephone in 1876 showed further complications in the use of telegraph lines for speech.

History (Continued)

Oliver Heaviside developed the theory and his work showed that telephone signals travelled best on lines made of two, large, copper wires mounted as widely-spaced as possible.

Heavyside noticed that the best transmission could be achieved if the distributed inductance of the line could be increased without affecting the other characteristics of the line.

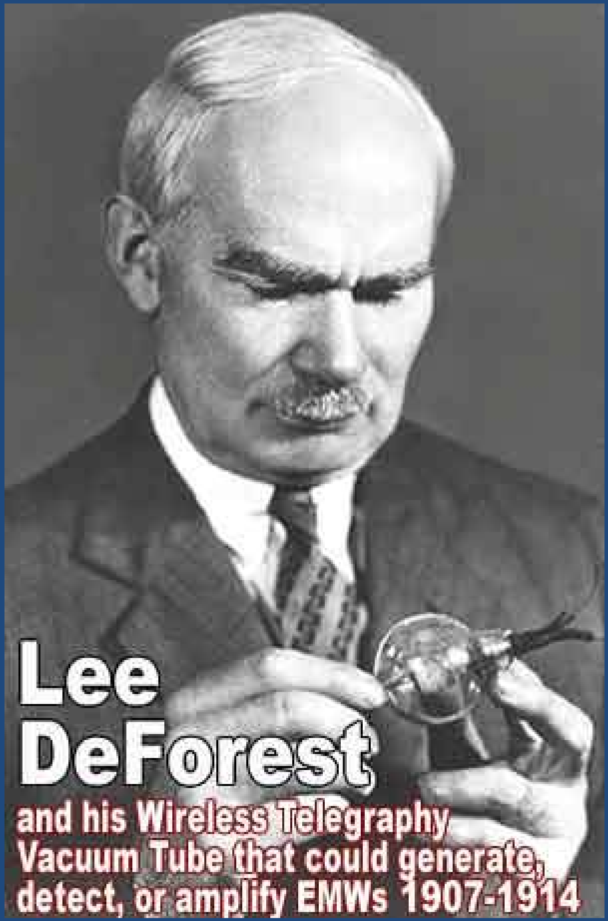
History (Continued)

As a practical solution “loading coils” were inserted into the lines every mile or so. This allowed smaller gauge wire to be used with a huge saving in costs.

In 1911 tests with carbon microphones , and no amplifiers, between New York and Denver showed that telephony over a distance of 2000 miles was not possible.

Lee De Forest’s triode valve of 1907 achieved a way round this. By 1915 transcontinental telephony using amplifiers and equalising networks was a reality.

Later Developments



Lloyd Espenschied and Herman A. Affel, [ca. 1949]
Inventors of coaxial cable, Lloyd Espenschied (left) and Herman A. Affel, examine sections of coaxial cable. In 1936, AT&T put in service the first coaxial cable for television use in New York City

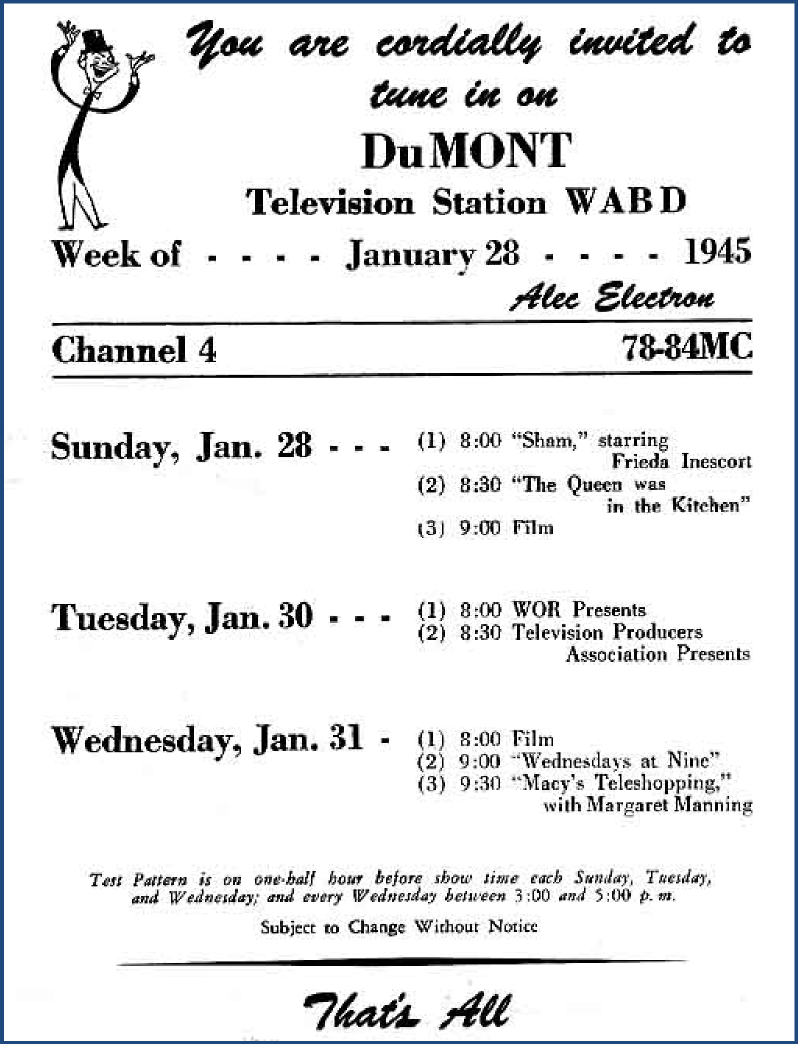
History (Continued)

Ironically, the loading of telephone lines earlier in the century had converted every telephone line into a low pass filter incapable of transmitting any frequency above 4kHz. And hence useless for carrier frequencies.

Wartime work in WW1 developed radio frequencies higher than 1 MHz. The growth of broadcast radio in the 1930s saw frequencies up to 30 MHz used for radio telephony and RF distribution by cables. Finally WW2 and the use of VHF and microwaves saw the application of transmission lines to much higher frequencies.

Two-conductor transmission lines remain, however, the basis for feeding antennas at HF.

TV Transmission



You are cordially invited to tune in on

DuMONT

Television Station WABD

Week of - - - - January 28 - - - - 1945

Alec Electron

Channel 4 **78-84MC**

Sunday, Jan. 28 - - -
(1) 8:00 "Sham," starring Frieda Inescort
(2) 8:30 "The Queen was in the Kitchen"
(3) 9:00 Film

Tuesday, Jan. 30 - - -
(1) 8:00 WOR Presents
(2) 8:30 Television Producers Association Presents

Wednesday, Jan. 31 -
(1) 8:00 Film
(2) 9:00 "Wednesdays at Nine"
(3) 9:30 "Macy's Teleshopping," with Margaret Manning

Test Pattern is on one-half hour before show time each Sunday, Tuesday, and Wednesday; and every Wednesday between 3:00 and 5:00 p. m.

Subject to Change Without Notice

That's All

In the United States before and after WW2 there was a lot of development in co-axial cable distribution for tv.

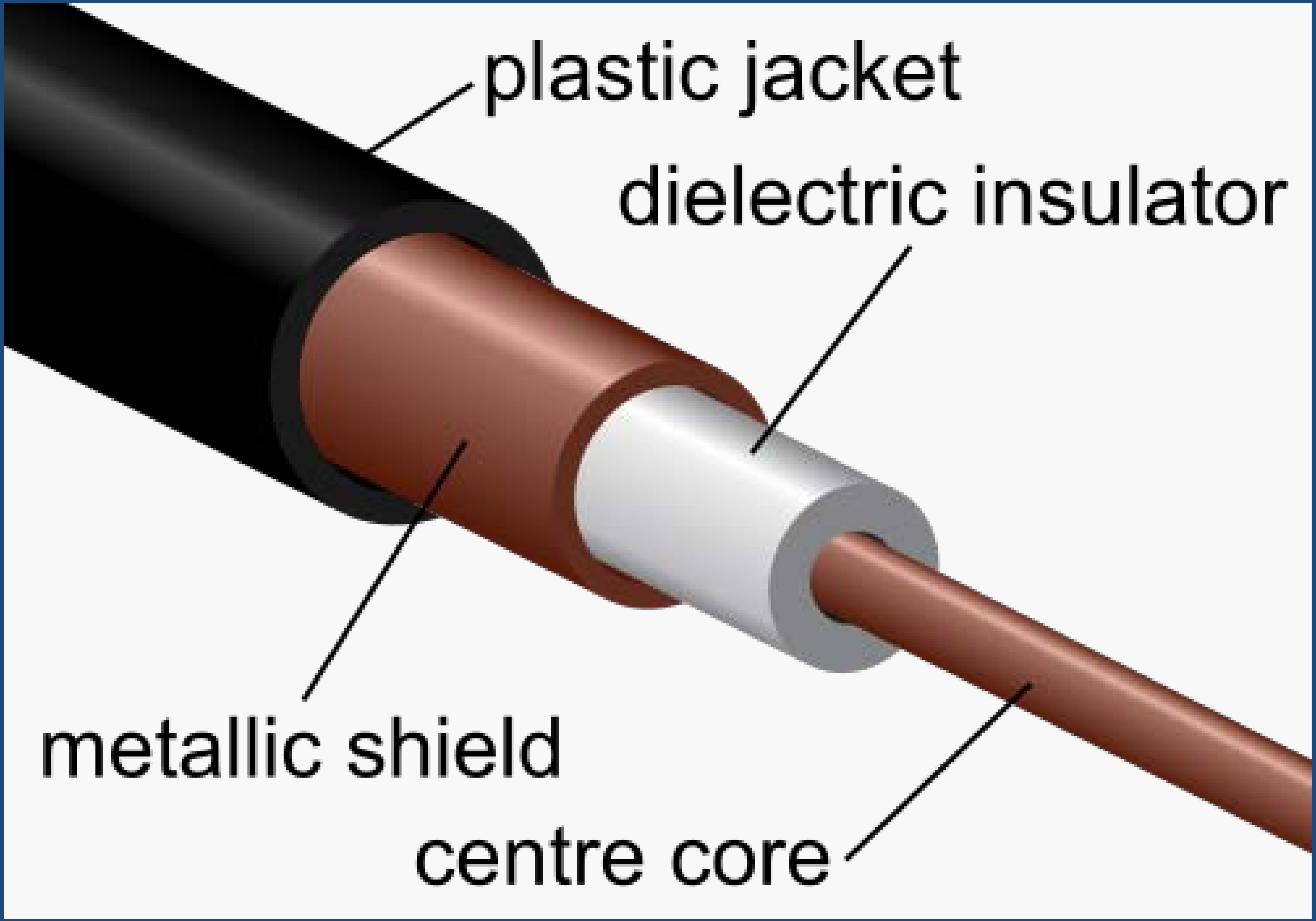
If you lived in a major city you could subscribe to a cable network for about an hour of tv each day.

Circuit Equivalent of the Line

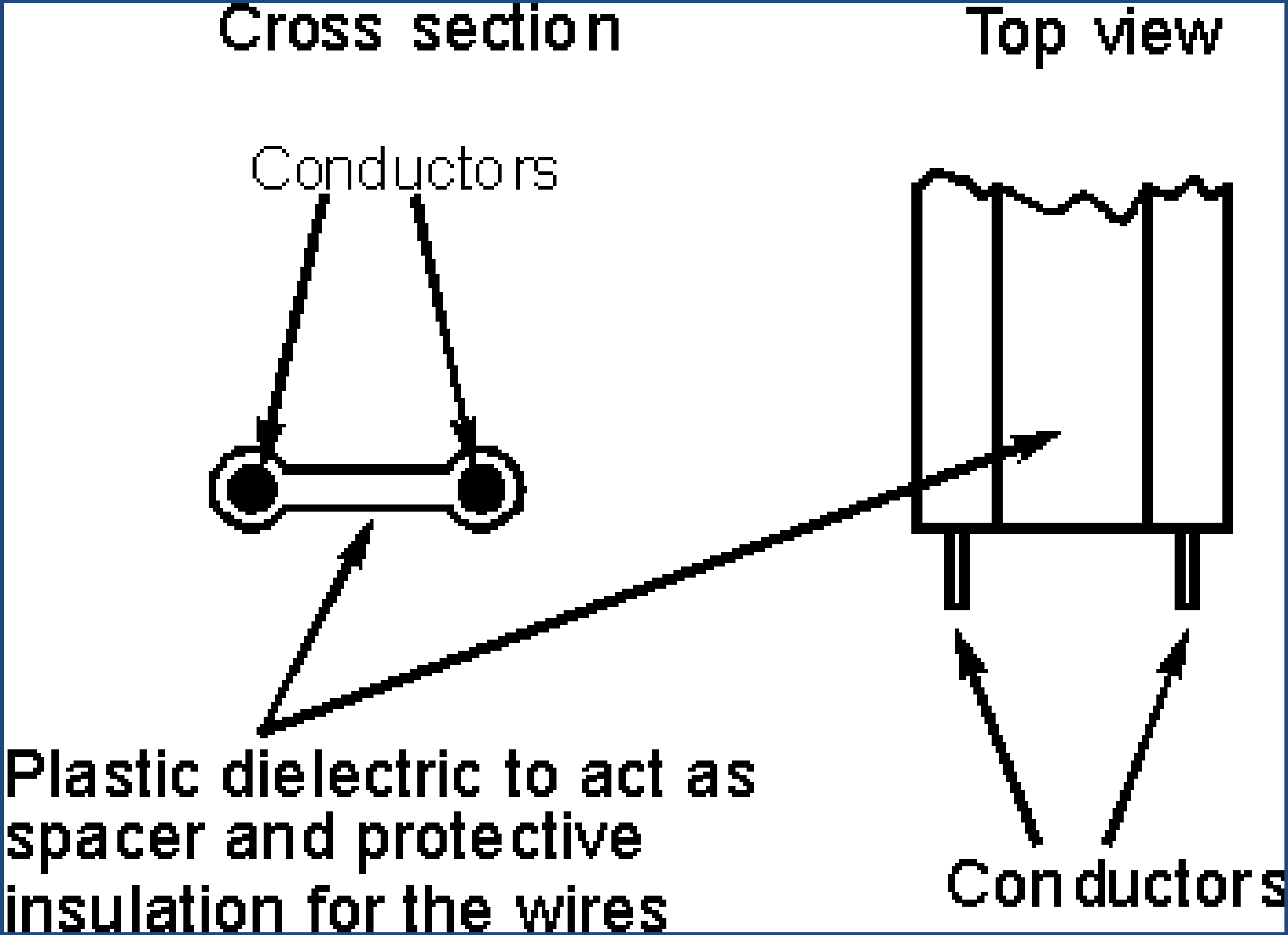
To reduce radiation and pick-up to a minimum, its two conductors must run very close together, so that the effects on or from one cancel out those on the other. One form is the parallel-wire feeder, another is the coaxial feeder in which one of the conductors encloses the other.

With either type – especially the coaxial – the closeness of the two leads causes a large capacitance between them. At very high frequency that means a low impedance. This, it might be supposed, would more or less short-circuit a long feeder so that very little power put in at one end would emerge at the other.

Co-Axial Cable



Twin Feeder



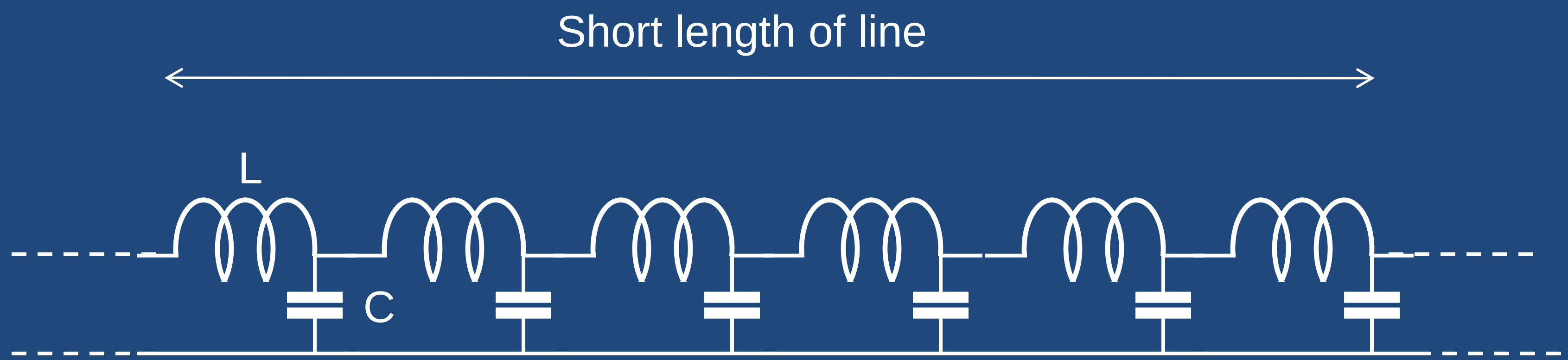
Cross section
Top view
Conductors
Plastic dielectric to act as spacer and protective insulation for the wires
Conductors

Circuit Equivalent of the Line

However each millimetre of cable as well as having a certain amount of capacitance in parallel also has, by virtue of the magnetic flux set up by the current flowing in it, a certain amount of inductance in series.

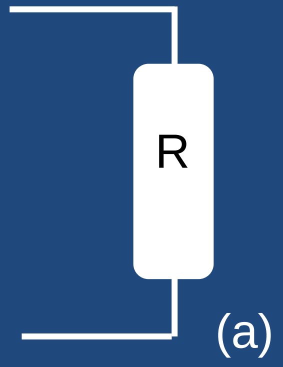
The following diagram shows a small length of transmission line. If the length is infinitesimally small then the values for L & C will also be small. Each inductance is of course contributed to by each line. However it makes no difference to show them all in one line. It is in series either way. Each L is accompanied by the same amount of C. In other words the ratio L/C is a constant.

Approximate Electrical Analysis



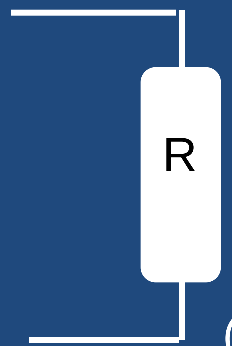
Approximate electrical analysis of a short length of transmission line.

Synthesis: Basic Cct. Elements

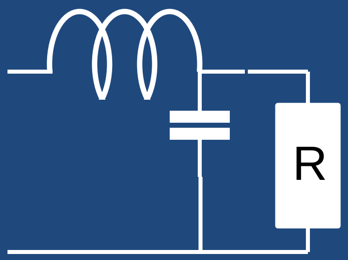


Considering the load at the end of the transmission line as a simple resistance R (a). If you measure the voltage across its terminals and the current through it you can work out the value of its resistance in Ohms.

Synthesis: Basic Cct. Elements



(a)



(b)

Now elaborate the load by adding a small inductance in series and a small capacitance in parallel (b). If the capacitance really is small, its reactance X_c will be much greater than R

Now, a relatively large reactance in parallel with a resistance can, with negligible error, be replaced by an equivalent circuit comprising the same resistance in series with a relatively small reactance.

Calling this equivalent series capacitive reactance X_c' we have $X_c'/R = R/X_c$ or $X_c' = R^2/X_c$

If we make X_c' = to X_L it will cancel out the effect of L giving the same reading as for R alone.

Basic Circuit Elements

X_L is of course $2\pi fL$

X_c is $1/2\pi fC$

$2\pi fL = R^2 \times 2\pi fC$

Or $L = R^2C$

Or, $R = \sqrt{(L/C)}$

Frequency does not come into this.

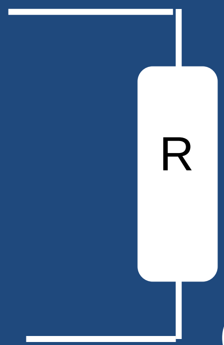
Except when it is very high L and C may have to become very small to fulfil he condition that X_c is much larger than R

Basic Circuit Elements

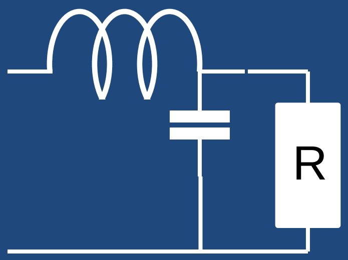
Assume that $\sqrt{(L/C)}$ is equal to the load resistance R in this case.

Further that R can be replaced by our combination of LCR as described above.

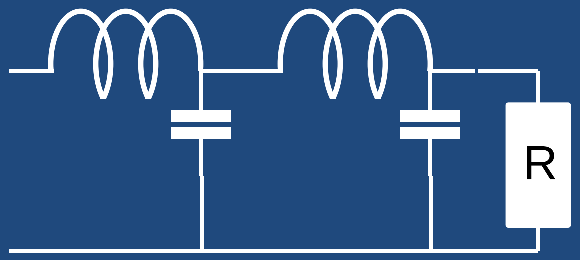
Synthesis: Basic Cct. Elements



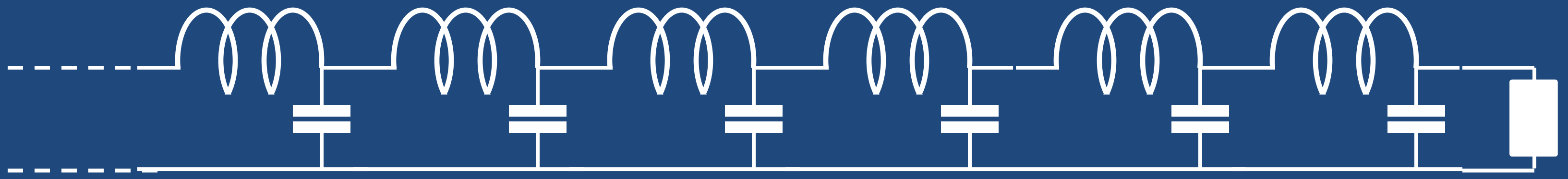
(a)



(b)



(c)



(d)

Then we can replace R as shown above in (b).

R can be replaced by another, identical LCR unit to the right and so on indefinitely. Each time you connect a meter to the load terminal it will show the same readings as R alone.

Basic Circuit Elements

The smaller you make L and C the closer the approximation becomes at all practical frequencies.

Ultimately you make L and C so small that they approximate more and more to the uniformly distributed inductance and capacitance in a transmission line.

In such a line the ratio of inductance to capacitance is the same for a millimetre as for a metre as for a kilometre..

Basic Circuit Elements

This leads, in the limit, to the conclusion that provided the load resistance R is equal to $\sqrt{(L/C)}$ then the length of the line makes no difference.

To the signal source or generator it is just the same as if the load were connected directly to its terminals.

(Exactly what you want as a feeder)

However......

A real-world transmission line will have conductors that have resistance. The line will absorb a certain amount of power.

A line made of thin wire will have correspondingly higher losses. So too will a pair separated by a poor insulator.

However for reasonable, practical applications the resistance can be neglected when making the analysis.

Characteristic Resistance

The next thing to consider is how to make the feeder fit the load so as to fulfil the necessary condition of

$$R = \sqrt{(L/C)}$$

$\sqrt{(L/C)}$ is obviously a characteristic of the line, and, to instruments connected across the line appears to be a resistance. For any particular line this is called its Characteristic Resistance. (If loss resistance is taken into account the expression is slightly more complicated than $\sqrt{(L/C)}$ and includes reactance.)

Strictly speaking the more comprehensive, and accurate term, is Characteristic (or Surge) Impedance Z_o

Load Resistance

If you feel that the resistance really belongs to the terminating load then remember that the terminating resistance can be replaced by another length of line. Ultimately the characteristic resistance can be defined as the input resistance to an infinitely long line.

Practically it is any resistance that can be fed through any length of line without making a significant difference to the generator.

Practical Feeders

L and C depend entirely on the spacing between the wires (tubes) and their diameters and the permittivity and permeability of the material between them.

So far as possible to avoid losses feeders are air-spaced. The closer the spacing the higher the C and the lower the L. (Because the magnetic field set up by the current in one wire more nearly cancels the field from the returning current in the other.)

Other things being equal one would expect a coaxial line to have a bigger C and lower R than parallel wire.

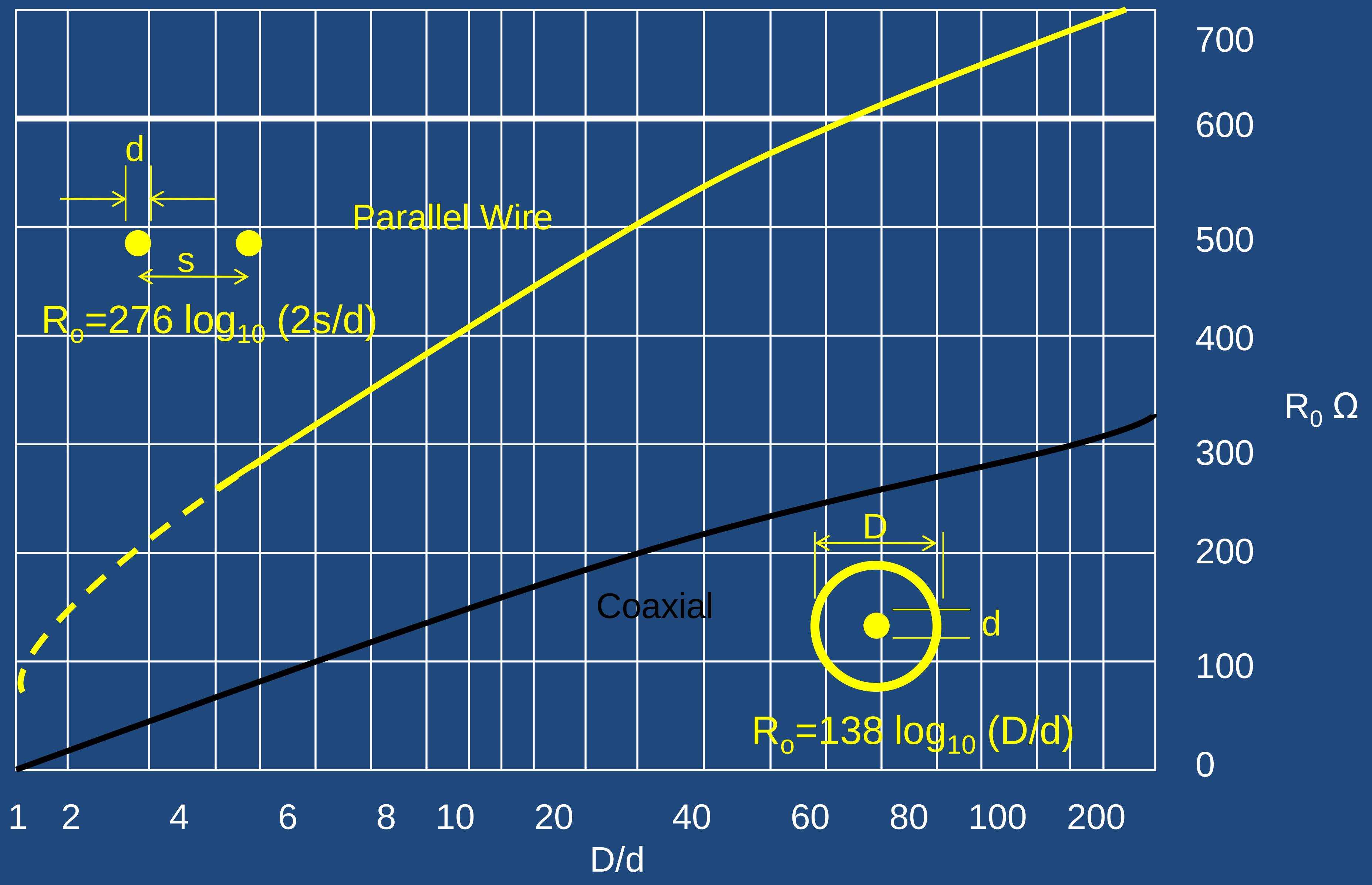
Practical Feeders (2)

For coaxial cable $R = 138\log_{10}(D/d)$

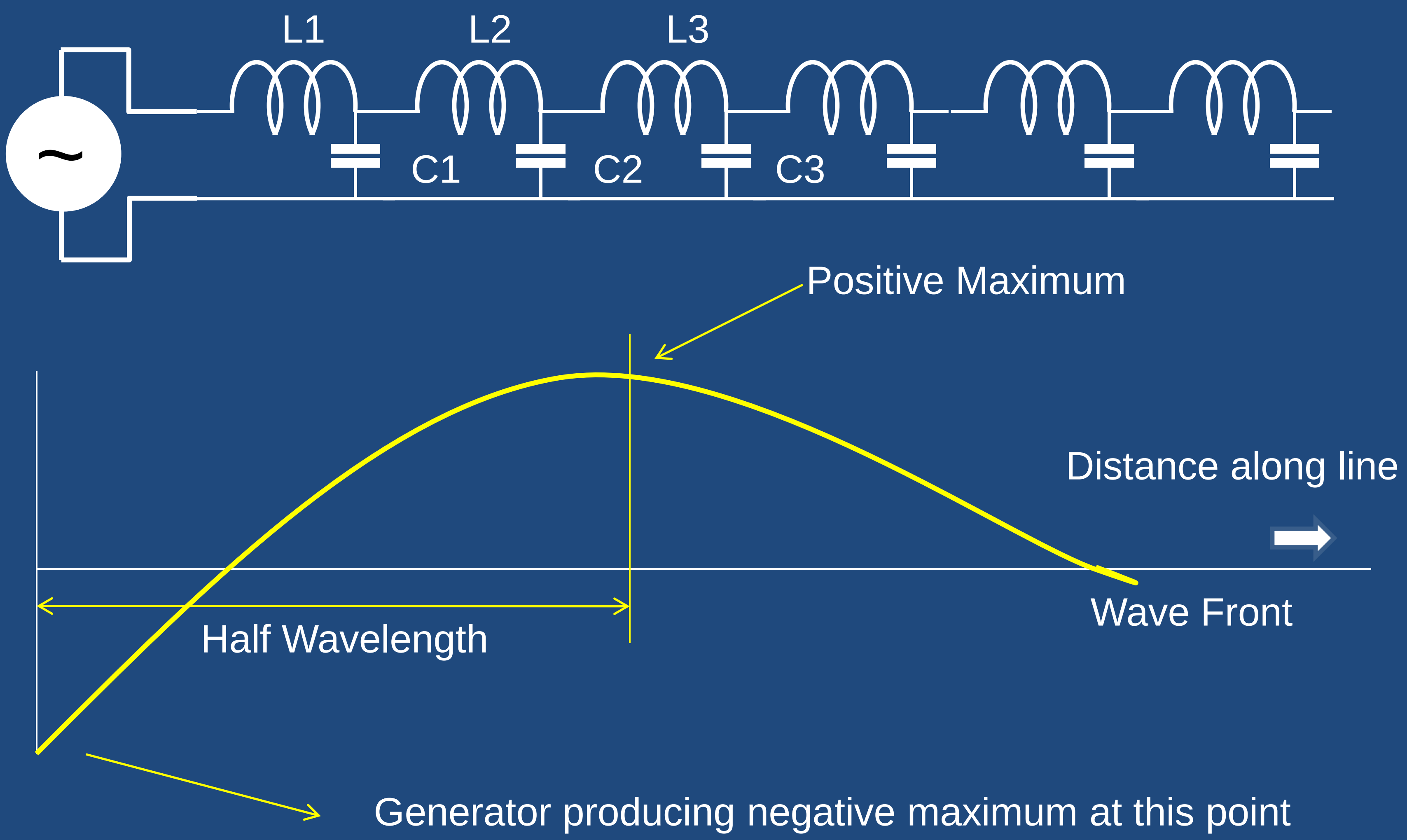
For parallel wires $R = 276\log_{10}(2s/d)$
(So long as s – the spacing – is at least 5 times greater than d the wire diameter.)

Experience has shown that feeder dimensions giving the lowest loss and within practical construction limits give an $R = 600\Omega$ for parallel wires and a value of $R = 80\Omega$ for coaxial cables

Characteristic Resistance R_o



Waves Along A Line



Waves Along a Line:

1. During the first half-cycle (say positive) current starts to flow into C1 charging it up. The inductance L1 prevents the voltage across C1 from rising to its positive maximum until a little later than the generator maximum.

2. The inductive inertia of L1 allows the charge on C1 to build a little before C2 and so on down the line.
3. Meanwhile the generator has gone onto its negative half cycle. This follows its predecessor down the line.

4. At its first negative maximum its voltage will be as shown in the previous slide. With a bit of imagination you can imagine the voltage wave flashing down the line like the waves on a long rope “waggled” up and down at one end.

5. The previous section showed that wherever the line is connected to a measuring instrument it appears as a resistance.

6. This means that the current is everywhere in phase with the voltage and hence power is being transferred along the line.

Velocity

The speed of the wave clearly depends on the inductance and capacitance of the line. The larger they are the longer the time each bit of the line takes to charge and the longer it takes for the current to build up.

The capacitance is increased by spacing the two wires more closely. The inductance is correspondingly decreased by bringing them closer together. As it happens the two effects largely cancel each other out so the speed of the wave is unaffected. The only result is that the ratio L/C and hence R_o is reduced.

Velocity

The only way of increasing C without reducing L is to increase the permittivity (ε) in the space between the conductors. Similarly changing the permeability (μ) affects the inductance.

The speed of the wave in the wire has been established as $1/\sqrt{\varepsilon\mu}$

The lowest values of ε and μ are those of free space (air).

With $\varepsilon = 8.854 \times 10^{-12}$ and $\mu = 1.257 \times 10^{-6}$ the speed comes out at 300,000,000 m/s

Velocity Continued.

There have to be solid supports for the conductors and sometimes the space is completely filled.

In practice the permeability is not changed but the permittivity is. Consequently the speed of the waves is slower.
If ε were increased to 4 the velocity of the waves would be about half that of light – but still pretty fast!

This means that the time needed to reach the far end of the line is short but finite. However short this time interval is there must always be a period between the power going into the line at the generator and appearing at the load.

During this time the generator is not in touch with the load.
The current pushed into the line by a given generator voltage is determined solely by R_o alone and not by whatever may be connected at the load end.

Power Transfer

During the brief interval when the generator does not "know" the resistance of the load it has to feed the characteristic resistance of the line controls the rate of power flow tentatively.

In accordance with the usual law the power flow will be a maximum if the characteristic resistance of the line R_o is equal to the generator resistance r.

Neglecting line loss the current and voltage reaching the load will be the same as that leaving the generator. So if the load turns out to be a resistance equal to R_o it will satisfy Ohms law and the whole of the power will be absorbed by the load as fast as it arrives.

Example.

For example if the characteristic resistance of the line is 500Ω and this is fed by a generator with an internal resistance of 500Ω and a voltage of 1000 Volts the terminal voltage is bound to be 500 Volts and the current 1 Amp until it reaches the far end – whatever may be there.

If the load resistance is 500Ω then the current will continue at 1 Amp everywhere.

Wave Reflection

But – supposing the load resistance is not 500Ω but 2000Ω instead?

According to Ohms law it is impossible for 500 Volts applied across a resistance of 2000Ω to cause a current of 1 Amp to flow. Yet 1 Amp is arriving – *so what does it do?*

Wave Reflection (1)

Part of the current, having nowhere to go starts back for home – the generator.

More scientifically it is reflected by the mismatch (the 2000Ω load)

The reflected current, travelling in the opposite direction can be regarded as opposite in phase to that arriving giving a total less than 1 Amp.

The comparatively high resistance causes the voltage across it to rise to above 500 Volts. This increase can be regarded as an in-phase reflected voltage driving the reflected current.

If half of the current were reflected leaving 0.5 Amps to go into the load resistance the voltage would be increased by half, making it 750 Volts

Wave Reflection (2)

A voltage of 750 and a current of 0.5 Amps would fit a load of 1500Ω – but not 2000Ω. This means that a bigger proportion has to be reflected – actually 60% giving 800 Volts and 0.4 Amps at the load.

In general if the load resistance is R then the fraction of the current and voltage reflected, called the *reflection coefficient* or *return loss* and denoted by ρ is:

$$\rho = (R-R_0)/(R+R_0)$$

Wave Reflection (3)

We now have 1Amp driven by 500 Volts travelling from the generator to the load and 0.6 Amps driven by 300 Volts returning to the generator.

The ratio of the reflected voltage to the reflected current must of course equal R_0

The combination of these two at the load terminals gives:

1 – 0.6 Amps at (500 + 300) Volts = 800 Volts

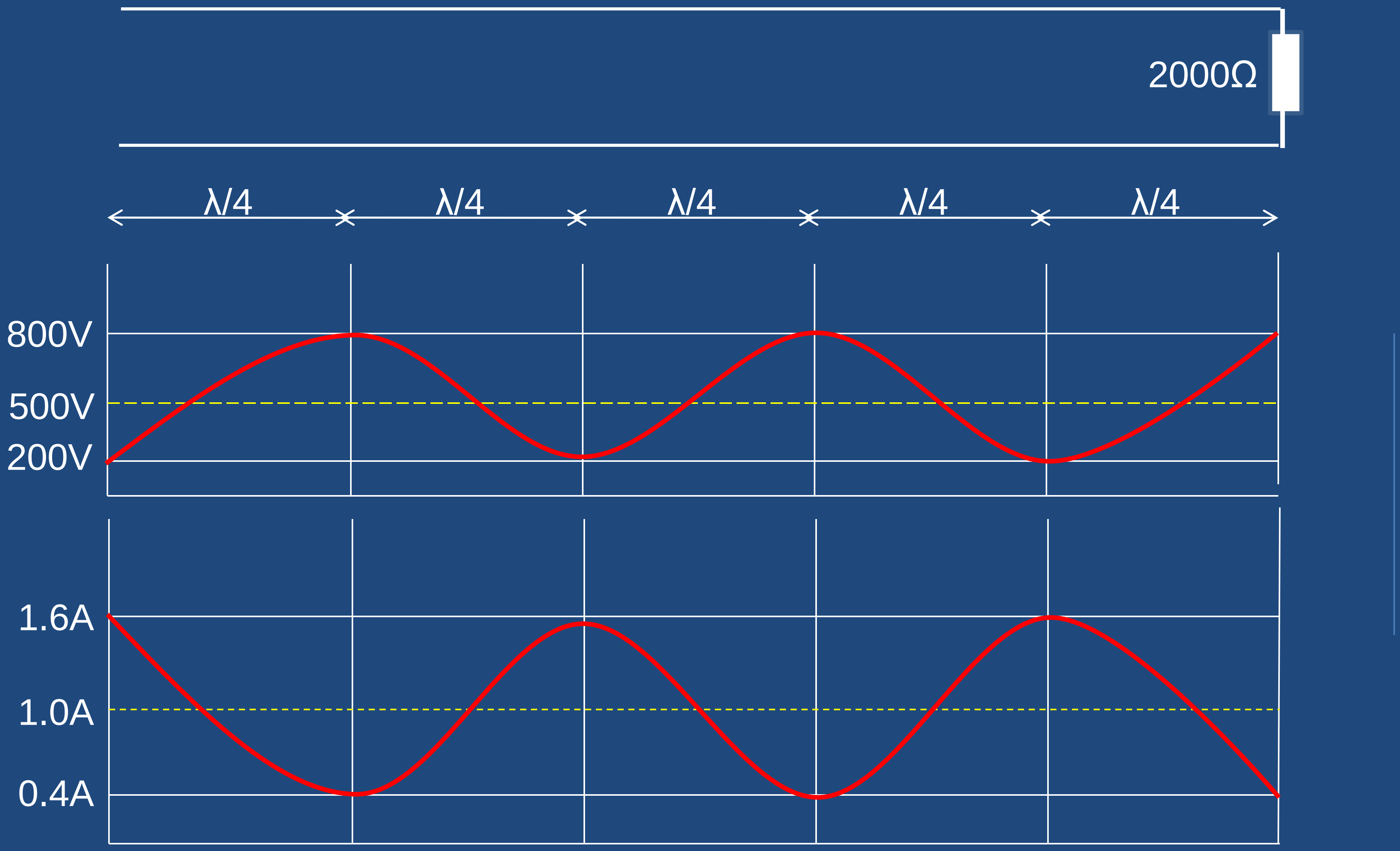
At other points along the line we have to take account of the phase lag. At 0.25λ the arriving and returning waves differ in phase by 0.5λ or 180 degrees compared to their relative phase at the load because the return journey has to be made over a 0.25λ distance.

Wave Reflection (4)

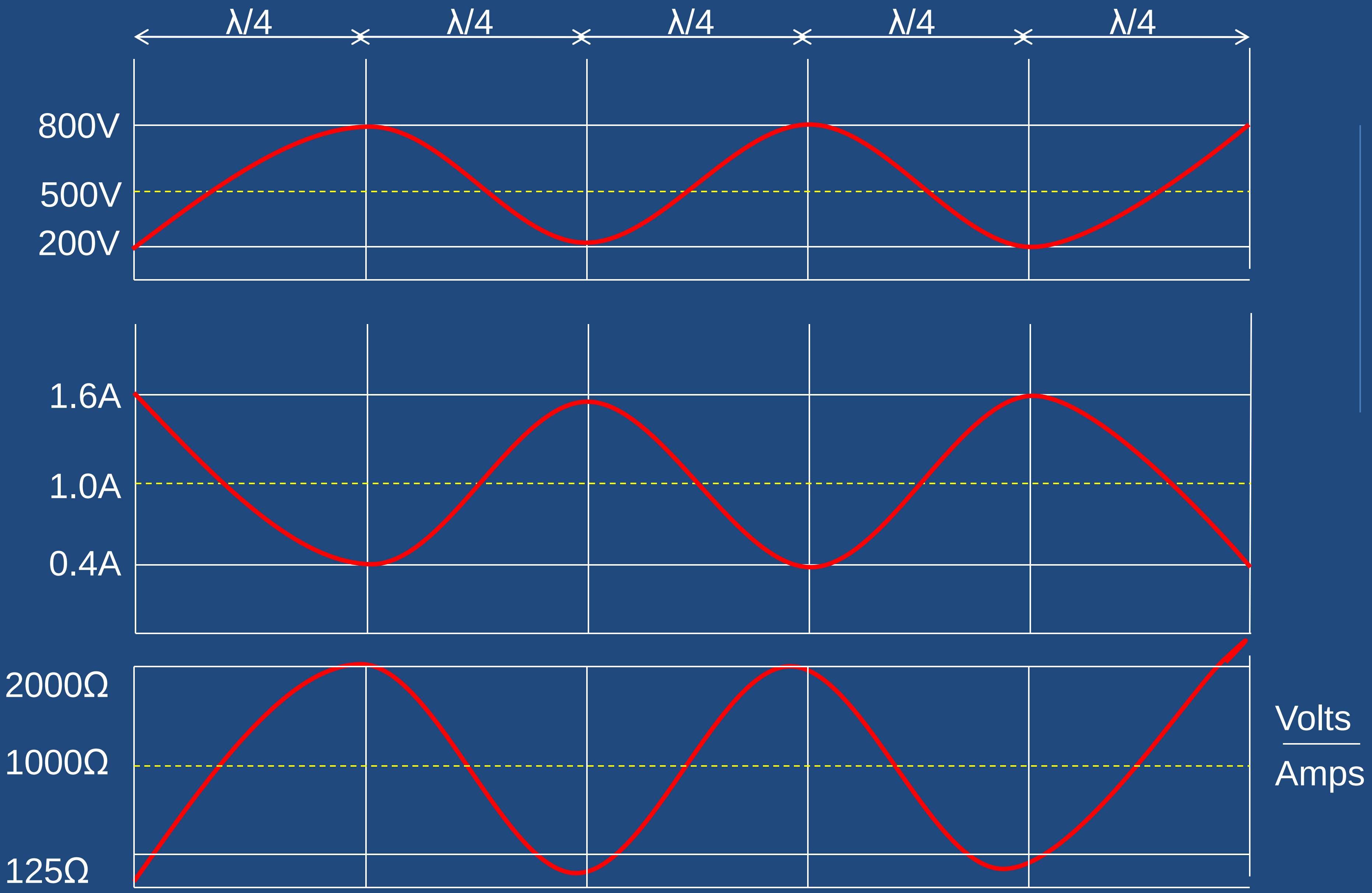
At this point the current is 1 + 0.6 Amps at 500 – 300 Volts = 200 Volts

At a point 0.125λ from the load the separation is λ/4 giving 1.16 A and 582 Volts. At intervals of λ/2 the waves come into step again.

Standing Waves1



Standing Waves



Standing Waves

Calculating the current and voltage along the line and plotting them point by point you can get the curves shown in the previous slides.

These are RMS values set up at the points continuously. Because of the wavelike nature of these curves they are called standing waves.

The dotted line on the graphs show the distribution when the line is matched.

The ratio of the maximum to the minimum current or voltage is called the standing wave ratio (swr). In the example it is 800:200 or a swr of 4.

The current swr is the same as the voltage swr. Usually only the vswr is given.

In general the swr is equal to R/R_0

Wave Reflection (5)

In due course the reflected wave arrives back at the generator. It is by reflection that the load makes itself felt by the generator.

If the 2000Ω load had been connected across the generator terminals the current would have been 1000/(500Ω + 2000Ω) or 0.4 Amps.
The terminal voltage would have been 2000 x 0.4 = 800 Volts and the power would have been 800 x 0.4 = 320 Watts.

This is exactly what the load at the end of the line is getting.

However the power that originally went out from the generator was determined by R_0 - the characteristic resistance of the line. And was 500 x 1 = 500 Watts.

The reflected power is 300 x 0.6 = 180 Watts.

The net outgoing power is thus 500 – 180 = 320 Watts –exactly what it would have been with the load directly connected.

Line Impedance Variations

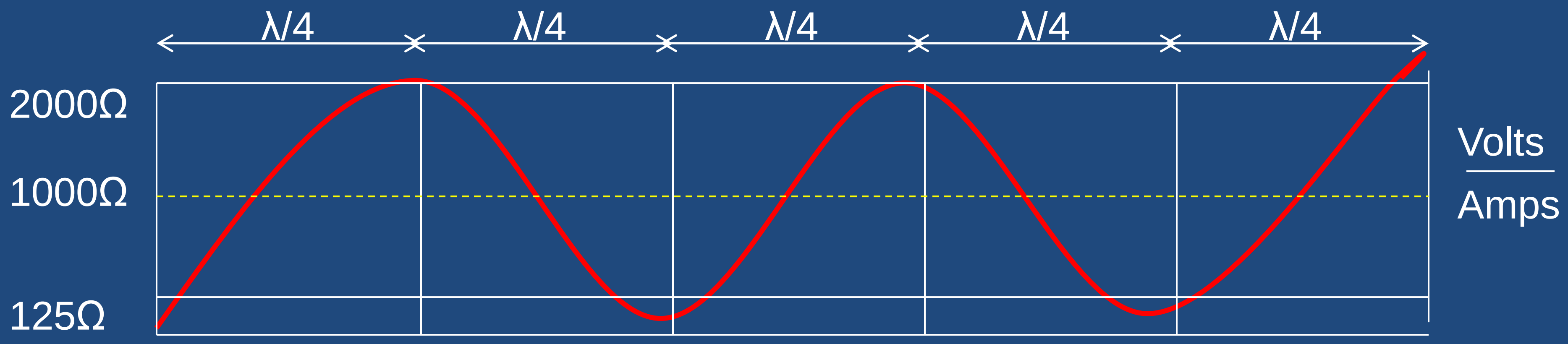
The power adjustment in this case is quite simple. The current and voltage situation at the generator is complicated by the current and voltage lag. It is not necessarily the same as at the load. Unless the line is an exact multiple of λ/2 the phase relationships are different.

If the line is an odd multiple of λ/4, eg 5λ4, then the current and voltage at the generator will be 1.6 A and 200 Volts. This makes 320 Watts. However the power loss in the generator is 1.6 x 1.6 x 500 = 1280 Watts.

At an 800 Volt point the current is 0.4 Amps and the loss in the generator were it connected here would be 0.4 x 0.4 x 500 = 80 Watts. So when there are standing waves the exact length of the line is very important.

Line Impedance Variations (2)

Connecting the generator at the point where the current and voltage are 1.6 Amps and 200 Volts is equivalent to connecting the generator to a load of 200/1.6 = 125Ω



This curve indicates indicates the impedance at any point when all the line to the left of that point is removed. The impedance depends not only on the distance along the line from the load but also on the characteristic resistance and the load impedance.

Line Impedance Variations (3)

From considering the travelling waves you can see that at λ/4 intervals along the line the phases of the voltage or current are either exactly the same as at the load or exactly opposite.

If the load is entirely resistive then the input resistance to the line is resistive.

At all other points the phases are such as to be equivalent to introducing reactance.

In the example the generator resistance was made equal to the R_0. If it were not then the situation gets more complex. Part of the power would be reflected back to the load and so on. The reflected power being smaller and smaller on each occasion.

The standing waves would be the resultant of all of these. Even very long lines settle down to a steady state quite quickly. However it is undesirable because it means that much of the power is being dissipated in the line and not the load.

Line Impedance Variations (4)

Mismatch at the generator end does not affect the swr but does affect the values of current and voltage reached.

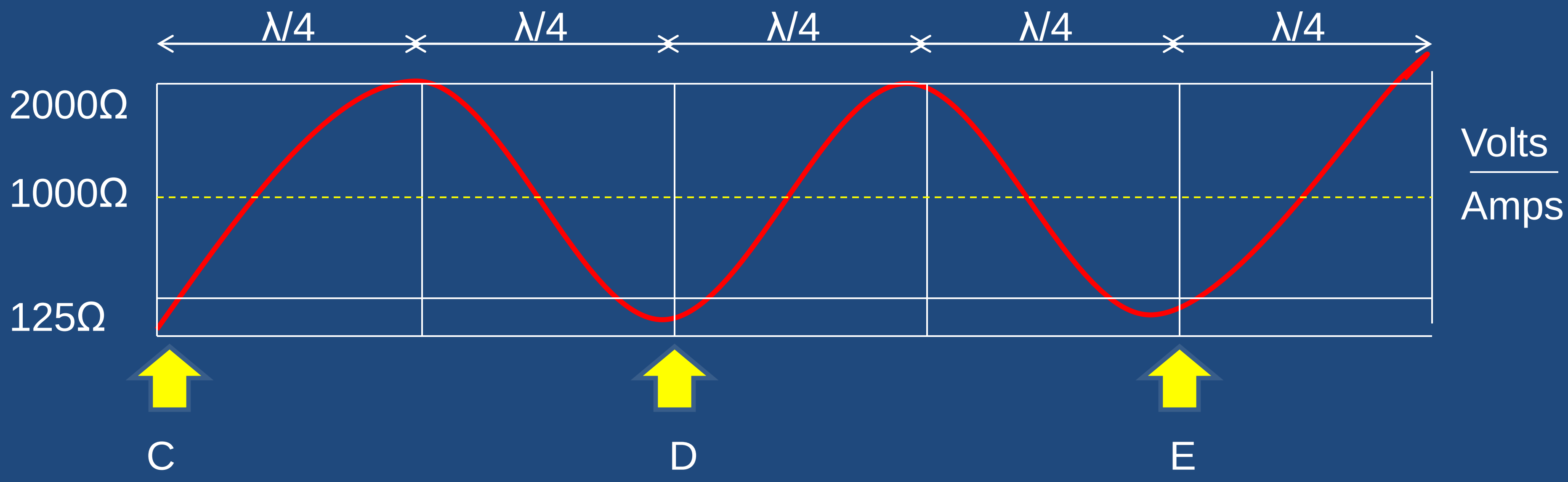
Perfect matching is that which enables maximum power to be transferred.

There may be good reason for deliberately mis-matching at the generator.

Using the figures we already have.

320 Watts and a loss of 80 Watts might be practically better than 500 Watts delivered but with a loss of 500 Watts

Quarter Wave Transformer



A generator of one impedance can be perfectly matched to a load of another impedance by suitably choosing the point of connection in the line. For instance a generator of 125Ω would be matched to the 2000Ω load if connected at C, D or E. The line then behaves as a 1:4 transformer. Points can be selected giving any ratio of 1:4 or 4:1 but except at the lettered points there will be reactance to be tuned out.

Quarter Wave Transformer (2)

You don't need to use the whole of the line. The maximum transformation is given by a length only Ω/4 long.

From our example the mismatch ratio was the same at both ends of 1:4
(125 to 500) and (500 to 2000).

In general terms if R_1 denotes the input resistance to the line (with the load connected) then:

$$R_1 \text{ is to } R_0 \text{ as } R_0 \text{ is to } R$$

This means that: $R_1/R_0 = R_0/R$ or $R_0 = \sqrt{R_1 R}$

This allows you to find the characteristic resistance of a quarter wave line needed to match two unequal impedances R_1 and R

Line Impedance Example

Suppose we wished to connect a 70Ω load to a 280Ω parallel wire feeder without reflection from the junction.

They could be matched by interposing a length of feeder to give a characteristic resistance of √(70 x 280) = 140Ω

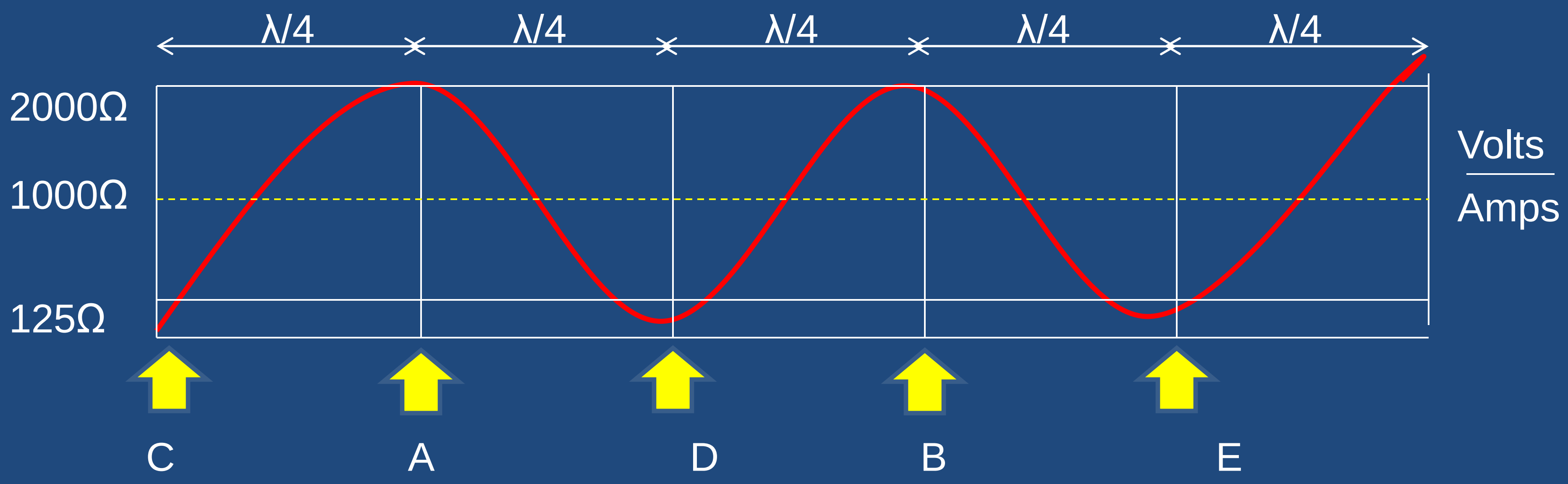
Parallel wire feeder would have excessively close spacing. However two lengths of 70Ω coaxial inserted at the load will effect the transformation.

Two sections of 70Ω coaxial line with their shields joined giving 140Ω

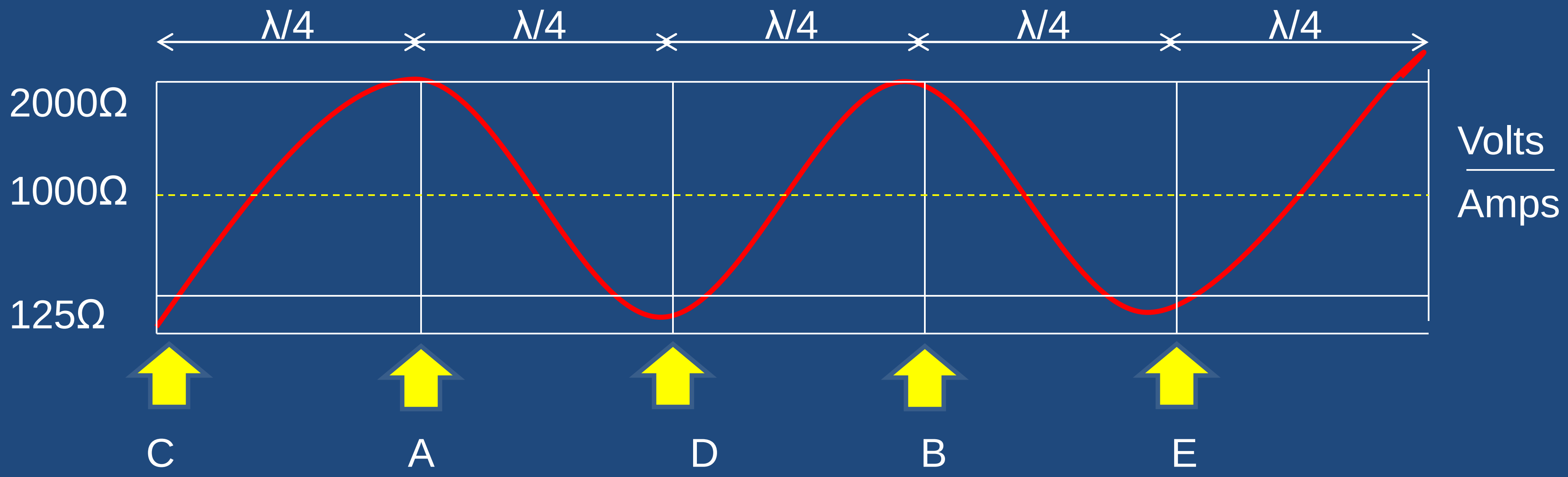
Fully Resonant Lines

What happens when the load resistance is either infinite or zero? (Open or short circuit)

With our example and an open circuit at the load then the current at the end would be zero and the voltage would rise to 1000 V double its amount across the matched load. (Remember it's a 1000 V generator)



Fully Resonant Lines (2)



This condition would be duplicated by the standing waves at A and B while at C, D and E the voltage would be 0 and the current 2 A.

The impedance curve would fluctuate between zero and infinity .

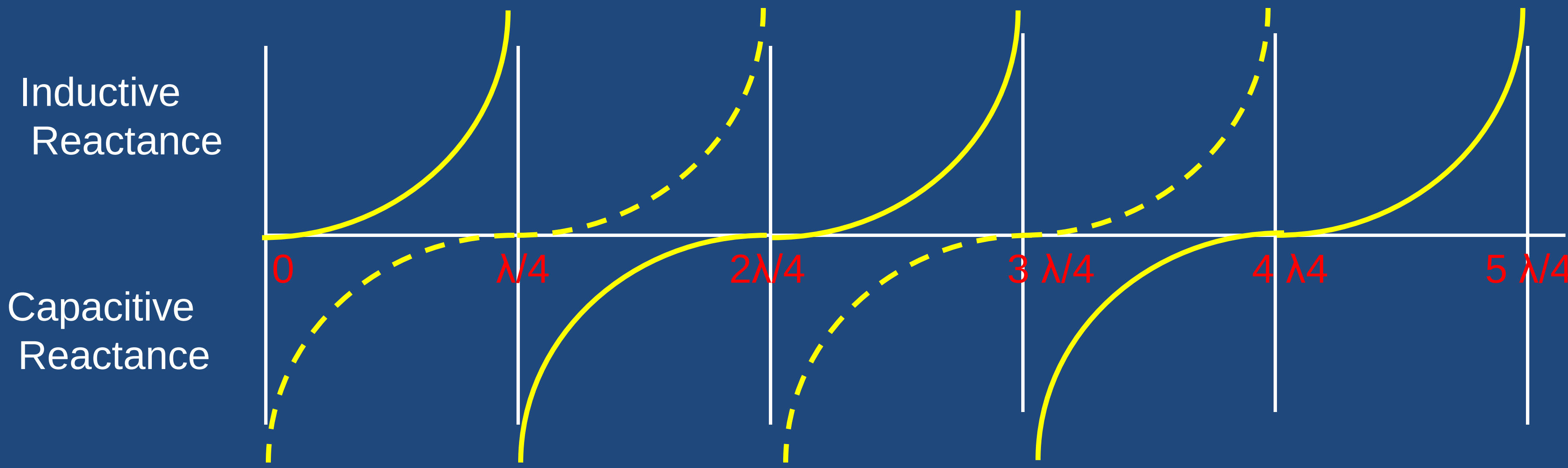
With a short-circuited line there could be no volts across the load and the current would be 2 Amps.

A short-circuited line is the same as an open-circuited line shifted a quarter of a wavelength along. Reflection in both cases is complete because there is no load to absorb the power.

Fully Resonant Lines (2)

If the generator resistance is very large or very small nearly all the reflected wave will itself be reflected back and so on. If the line is of such a length that the voltage and current maximum points coincide with every reflection then the voltage and current will build up to high values at these maximum points. If the transmitter is a powerful one then dangerously high voltages and currents will occur. This is similar to high Q resonant circuits.

Reactance of Open & Short Circuited Lines



When the length of a short-circuited or open-circuited length of feeder is a whole number of quarter wavelengths the input impedance is approximately zero or infinity.

An odd number of quarter wavelengths gives opposites at the end – infinite resistance if the other end is shorted and vice-versa. An even number of quarter wavelengths gives the same at each end.

In between the impedance is pure reactance. Each side of a node or anti-node there are opposite reactances. A short length of line can provide any value of inductance or capacitance. (This is useful for stubs.)

END